The Effect of Bank Angle and Weight on the Minimum Control Speed V_{MCA} of an Engine-out Multi-engine Airplane

Harry Horlings[1], Flight Test Engineer
Avio*Consult*, Leiderdorp, The Netherlands

1. Introduction

1.1. Engine failures on multi-engine airplanes continue to result in fatal accidents all around the globe quite frequently, not only during takeoff but also during climb, cruise, and approach for landing. Most often, control is lost because the pilots maneuver the airplane after engine failure at low speed. Aviation Regulations (Refs 1 and 2) require a "*Minimum Control speed, a calibrated airspeed at which, when the critical engine is suddenly made inoperative, it is possible to maintain control of the airplane with that engine still inoperative, and thereafter maintain straight flight at the same speed with an angle of bank of not more than 5 degrees*" (away from the failed engine). The Minimum Control speed must be published in the Limitations Section of the Airplane Flight Manual. Minimum Control speed is abbreviated V_{MC} in older publications, however, more V_{MC}'s are required to be determined: Minimum Control speed on the Ground (V_{MCG}), Minimum Control speed approach and Landing (V_{MCL},), and Minimum Control speed approach and Landing with two engines inoperative (V_{MCL2}). This paper is about V_{MCA}, the V_{MC} in-the-Air or Airborne.

1.2. Manufacturers and flight schools publish and teach V_{MCA}, but do not include the flight conditions that are required for the published V_{MCA} to be valid, such as straight flight only while maintaining a small bank angle away from the failed engine to keep V_{MCA} low and to reduce the sideslip, hence drag as much as possible for maximum climb performance. Many pilots learn that turning (at low speed) while an engine is inoperative is preferred only into the good engine' side, because the control margin would then be a bit larger. But is this true? What is the effect of a change of bank angle on V_{MCA}, hence on airplane control? Can turns indeed be safely made at airspeeds as low as V_{MCA} or at low takeoff speeds when one engine is inoperative and the thrust of the opposite engine is set to maximum?
The negative answer is given in paper *Airplane Control and Analysis of Accidents after Engine Failure* (Ref. 3) and in other papers and a YouTube video, that are downloadable from and via the website of Avio*Consult*, the target audience of which are flight instructors, pilots, and accident investigators.

1.3. This paper proves by analysis of the stability derivatives of an airplane, that when the thrust distribution on an airplane is asymmetrical, turns can only be safely made at airspeeds that are much higher than the V_{MCA} and the therewith derived takeoff safety speeds that are published in the limitations and performance sections of an Airplane Flight Manual. The effect of bank angle and weight on V_{MCA} (and on the takeoff or go-around speeds), as presented and discussed in the above referenced paper, was calculated using the method presented below, which is taught at Test Pilot Schools (Refs 4, 5 and 6) and used by Experimental Test Pilots and Flight Test Engineers to predict the static V_{MCA} during preparation for experimental flight tests to determine V_{MCA} in-flight for new or modified airplanes to be aware of the required control inputs during decreasing the airspeed and of the control limitations that might be encountered during the tests. Airplane design engineers already used this method during designing the airplane and sizing the vertical tail with rudder and ailerons (Ref. 7). These calculations can also be used to calculate and show the effect of bank angle and weight on V_{MCA} in a format that will be understood by (airline) pilots and accident investigators, which is the objective of this paper. The most recent version can be downloaded from the Downloads page of the website of Avio*Consult*[1].

---

[1] Owner Avio*Consult*, Lt-Col RNLAF ret., BSc, graduate Flight Test Engineer USAF Test Pilot School, Class (19)85A; email: info@avioconsult.com or avioconsult@gmail.com, website: https://www.avioconsult.com.

Copyright © 2007 – 2023 Avio*Consult*

2. Forces and moments acting on an airplane

2.1. An airplane in-flight has six degrees of freedom, it can accelerate and move forward and aft, sideways left and right, up and down, and also rotate about three axes. The translational and rotational motions are not only caused by external forces that act on the airplane, but also due to aerodynamic forces and moments caused by control inputs of rudder, aileron, and elevator in the three axes and by the propulsion systems and its malfunctions, or other failure states of the airplane itself. These forces and moments are vector quantities that have a direction and a magnitude. A *force* moves a body in the direction of the force, a *moment* (is a force × its perpendicular distance – called arm – from the center of gravity) produces a rotation of a body around an axis. It is unavoidable to describe forces and moments when analyzing airplane control when an engine fails or is inoperative.

2.2. There are several coordinate systems in use to describe forces and moments that act on an airplane. Pilots are used to explain turns using the centripetal force, being the horizontal component of the lift generated by the wings in the flat earth referenced coordinate system (that is fixed to the aircraft's center of mass – the center of gravity), as shown in Figure 1. The lift of the wings and the drag are, by definition, perpendicular resp. parallel to the velocity vector (not shown). As shown in Figure 1, the weight of the airplane acts along the vertical axis (towards the center of the earth) and hence has no side force (lateral) component in this axis system. The vertical component of the Lift (L·cos ϕ) must equal the Weight to maintain level flight. There is nothing wrong with using this axis system for calculating turns, as long as the airplane is healthy and the flight is coordinated. But in this coordinate system, the lateral-directional forces, and the rolling and yawing moments about the horizontal and vertical (earth) axes, cannot easily be analyzed. These forces and the resulting moments though, are needed during the analysis of the controllability after engine failure, which is the subject of this paper. Therefore, in order to describe and explain the controllability of an airplane after a propulsion system malfunction, another coordinate system is a lot easier to use: the *body-fixed coordinate system* (Figure 2).

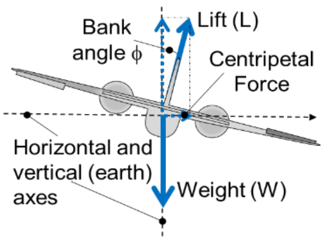

Figure 1. Centripetal force as component of Lift in flat earth referenced and inertial coordinate systems.

2.3. As shown in Figure 2, the three body axes are fixed to the airframe, hence move with it and originate also in the center of gravity; the positive x-axis points toward the front, the positive z-axis points down the bottom and the positive y-axis points to the right, parallel to the wings; all three axes are perpendicular to each other.

Figure 2 shows that the Lift generated by the wings acts along the z-body axis and therefore has no side force, no lateral component in the y body axis. The Weight however, acts towards the center of the earth and does have components in the x, y, and z body axes. The Lift must equal the z-axis component of the weight (W·cos ϕ) to be able to maintain level flight. The side force component of the weight (W·sin ϕ) provides for a side force in the direction of the y-body axis (similar to the centripetal force above). The body-axis system allows the fairly easy analysis of the effects of rudder and aileron deflections δ_r and δ_a, of sideslip angle β, of weight W, of bank angle ϕ and of asymmetrical thrust T on the lateral and directional forces and moments that act on the airplane, reason why this coordinate system is also used by aeronautical engineers during designing the airplane and by experimental test pilots and flight test engineers to prepare (engine-out) flight testing.

In this paper, only the most relevant forces and moments will be discussed, that act in the body-fixed axes on an airplane and that play a significant role for controlling an airplane laterally and directionally after engine failure.

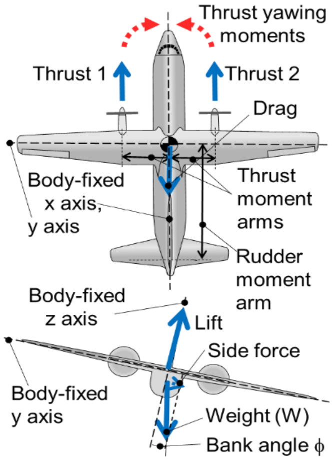

Figure 2. Forces and moments, including side force as component of weight, in the body-fixed axis system.

3. Sign convention

3.1. Several sign conventions are in use in aviation. The sign convention that was used in Ref. 4, the source document of this paper, is used in this paper and is presented in Table 1.

Control Surface	Symbol	Sign	Direction	Controller Deflection	Symbol	Sign	Direction	Change
Elevator	δ_e	−	Trailing edge up	Pitch stick/column	δ_{es}	+	aft	+ΔM, +Δα
Aileron	δ_a	+	Left dn, right up	Lateral stick/wheel	δ_{as}	+	right	+ΔL, +Δϕ
Rudder	δ_r	+	Trailing edge left	Rudder pedal	δ_{rp}	+	right	+ΔN, −Δβ
Sideslip	β	+	Wind in right ear (with left rudder -δ_r)					

Table 1. Sign convention used in this paper.

A few parameters in words: A control stick defection or wheel rotation (δ_{as}) to the right (+) results in a positive rolling moment (L) and a positive (+) incremental change (Δ) of bank angle ϕ, both to the right. A right rudder pedal input (δ_{rp}) (+) results in a positive incremental change of the yawing moment (N) to the right, but in a negative change of sideslip angle β (−) (wind in left ear); the rudder direction is trailing edge right, which has a negative sign. To avoid misunderstanding, the sign convention is repeated in the legend of the charts.

4. The relation between airspeed, bank angle and weight

4.1. The motions of an airplane can be described in equations of motion. The stability derivatives in these equations describe the change in aerodynamic forces and moments with changes of the specified variables such as control inputs or atmospheric disturbances. Stability derivatives in non-dimensional form are coefficients (C_x) in the equations that can be calculated, or determined during flight-testing for a specific configuration and flight regime, for instance for an aft center of gravity (cg), flap position or speed range. Many more variables have effect on V_{MCA}, refer to Ref. 3 for details. For describing the effect of bank angle and weight on V_{MCA}, the lateral-directional equations of motions (lateral and directional moments and side forces) are of most importance.

4.2. The lateral-directional stability derivatives used in this paper and their meaning are:

Symbol	Meaning	Symbol	Meaning
$C_{\ell\beta}$	Rolling moment due to sideslip angle	$C_{n\delta r}$	Yawing moment due to rudder deflection
$C_{\ell\delta a}$	Rolling moment due to aileron deflection	C_{no}	Yawing moment due to configuration asymmetry
$C_{\ell\delta r}$	Rolling moment due to rudder deflection	$C_{y\beta}$	Side force due to sideslip angle
$C_{\ell o}$	Rolling moment due to config. asymmetry	$C_{y\delta a}$	Side force due to aileron deflection
$C_{n\beta}$	Yawing moment due to sideslip angle	$C_{y\delta r}$	Side force due to rudder deflection
$C_{n\delta a}$	Yawing moment due to aileron deflection	C_{yo}	Side force due to configuration asymmetry

Table 2. Lateral-directional stability derivatives used in this paper.

4.3. In Ref. 4 simplified equations are derived and also presented in Refs 5 and 6 and below. These equations were simplified for unaccelerated 1 g, constant heading flight with zero roll, yaw and sideslip rates, i.e., for a static equilibrium, the result of which is included in § 4.4 below. These simplified equations allow the calculation and plotting of the required rudder (δ_r) and aileron (δ_a) deflections and the resulting sideslip angle (β) of a multi-engine airplane for maintaining equilibrium flight at several weights (mg = W), calibrated airspeeds (q – dynamic pressure), asymmetrical thrust levels (N_T – at various altitudes) and (small) bank angles (ϕ) into and away from the inoperative engine(s).
During flight-testing in accordance with FAA and EASA Flight Test Guides (Refs 8, 9 and 10), the dynamic V_{MCA} (after a sudden engine failure) is determined as well as the static V_{MCA} that applies during the remainder of the flight when one engine (the critical engine – that returns the highest V_{MCA} after failure of any engine) is inoperative and the engine opposite of the inoperative engine is set to the produce the maximum thrust that the pilot can set from the cockpit with the throttles. Other factors that have influence on V_{MCA} need to be at their worst-case value for V_{MCA}, such as an aft cg (for shortest rudder moment

arm) and lowest weight (for smallest effect of weight when banking), for the published V_{MCA} to be the highest, the worst-case V_{MCA}, which is of course safest to publish. Following the test at a safe altitude, the V_{MCA} data are extrapolated to sea level (or other altitudes) for publication in the limitation section of the Airplane Flight Manual. The dynamic and static V_{MCA}'s, the influencing factors and the flight-tests are described in detail in Ref. 3; the dynamic V_{MCA} is usually not higher than the static V_{MCA}.

4.4. For the purpose of this paper, only the static V_{MCA} is calculated. The simplified lateral-directional equations for the static case, copied from Ref. 4 (page 32-16) and confirmed in Ref. 5 are:

$$C_{y_\beta}\beta_{trim} + C_{y_{\delta_a}}\delta_{a_{trim}} + C_{y_{\delta_r}}\delta_{r_{trim}} = \frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}$$

$$C_{l_\beta}\beta_{trim} + C_{l_{\delta_a}}\delta_{a_{trim}} + C_{l_{\delta_r}}\delta_{r_{trim}} = \frac{-L_T}{\bar{q}Sb} - C_{l_0}$$

$$C_{n_\beta}\beta_{trim} + C_{n_{\delta_a}}\delta_{a_{trim}} + C_{n_{\delta_r}}\delta_{r_{trim}} = \frac{-N_T}{\bar{q}Sb} - C_{n_0}$$

These three equations define the static side forces (y), and the lateral (ℓ) and directional (n) moments respectively (ℓ and n for small perturbations). Bank angle ϕ shows up in the side force equation only, but control inputs of rudder (δ_r) and aileron (δ_a), and the sideslip angle β have effect in all three equations, reason why the system of all three equations must be solved to isolate the control inputs and sideslip, rather than only one of them. They are so called simultaneous equations.
In the equations, dynamic pressure ($q = ½ \rho V^2$, in which V = airspeed), weight (mg), thrust (N_T), and bank angle (ϕ) are the input variables. Fixed are the Coefficients, wing area (S) and span width (b). The Coefficients (C) describe the effects of control inputs and sideslip on forces and moments that act on the airplane for a given configuration. For instance, the directional coefficient $C_{n\beta}$ (also called weathercock stability) describes the change in yawing moment n with changes in sideslip angle β; the lateral $C_{\ell\beta}$ describes the rolling moment ℓ due to sideslip angle β of this airplane. F_y is the sum of the side forces in the Y_b (body) axis due to lost thrust, spillage drag and lift, L_T is the rolling moment due to thrust (such as propulsive lift by a propeller or engine alignment), both of which are considered zero for the turbojet airplane used in this paper.
A configuration asymmetry such as caused by the separation of an engine from a wing, causes a configuration asymmetry and might affect all three equations through the C_{y0}, $C_{\ell 0}$, and C_{n0} coefficients.
Trim as used in this paper means the trim setting plus the manual control inputs in all three axes if the available trim controls are not adequate.

4.5. In deriving these equations, the small angle assumption was used; cos θ = 1 and sin $\phi = \phi$ (in radian measure) because pitch angle θ is usually less than 15 degrees and bank angle ϕ is maximum 5 degrees for determining V_{MCA}.

4.6. As there are four motion variables in these three equations (ϕ, δ_a, δ_r and β), many states of equilibrium are possible, but only the cases in which ϕ = 0° – 5°, and β = 0° are of most interest for the prediction of V_{MCA} and for the control inputs required (up to maximum deflection available) after engine failure to maintain an equilibrium of forces and moments, hence to prevent the loss of control.
Regulations (Refs 1, 2) allow a maximum of 5° of bank for determining V_{MCA}, which is usually already used during sizing the vertical tail with rudder. Zero bank angle ϕ (wings level) is easy to fly (in instrument meteorological conditions), but a zero-sideslip angle β (resulting from a small bank angle away from the failed engine) causes the total drag of the airplane to be minimal, which is favorable to the remaining climb performance after engine failure, especially for small twin-engine airplanes.

Copyright © 2007 – 2023 AvioConsult

4.7. The equations in § 4.3 above were solved for δ_r, δ_a and β using Cramer's rule, the results of which are the following matrices:

$$\Delta = \begin{vmatrix} C_{l_\beta} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ C_{n_\beta} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \\ C_{y_\beta} & C_{y_{\delta_a}} & C_{y_{\delta_r}} \end{vmatrix}$$

$$\beta_{trim} = \frac{\begin{vmatrix} \frac{-L_T}{\bar{q}Sb} - C_{l_0} & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ \frac{-N_T}{\bar{q}Sb} - C_{n_0} & C_{n_{\delta_a}} & C_{n_{\delta_r}} \\ \left(\frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}\right) & C_{y_{\delta_a}} & C_{y_{\delta_r}} \end{vmatrix}}{\Delta} \text{ radians}$$

$$\delta_{r_{trim}} = \frac{\begin{vmatrix} C_{l_\beta} & C_{l_{\delta_a}} & \frac{-L_T}{\bar{q}Sb} - C_{l_0} \\ C_{n_\beta} & C_{n_{\delta_a}} & \frac{-N_T}{\bar{q}Sb} - C_{n_0} \\ C_{y_\beta} & C_{y_{\delta_a}} & \left(\frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}\right) \end{vmatrix}}{\Delta} \text{ radians} \qquad \delta_{a_{trim}} = \frac{\begin{vmatrix} C_{l_\beta} & \frac{-L_T}{\bar{q}Sb} - C_{l_0} & C_{l_{\delta_r}} \\ C_{n_\beta} & \frac{-N_T}{\bar{q}Sb} - C_{n_0} & C_{n_{\delta_r}} \\ C_{y_\beta} & \left(\frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}\right) & C_{y_{\delta_r}} \end{vmatrix}}{\Delta} \text{ radians}$$

Figure 3. Lateral-directional equations of motion in matrix format.

4.8. In these matrices, the rolling moment L_T is zero, because the engines do not augment the lift of the wings of the sample airplane used in this paper. The yawing moment due to engine failure N_T is calculated as follows: the engine-out failure state is equivalent to adding a negative thrust vector, in this example −17,000 lb of the failed engine #1, which is located at −45 ft (to the left) on the Y body axis (Ref. 4). Then the added thrust yawing moment:

- For one engine (#1) inoperative (n−1) for calculating the standardized V_{MCA}:
  $N_T = \ell_x \cdot F_y - \ell_y \cdot F_x = 0 - (-45 \text{ ft}) \times (-17{,}000 \text{ lb}) = -765{,}000$ ft-lb.
- For two engines (#1 and #2) inoperative (n−2), for calculating V_{MCA2}, the minimum control speed with two engines inoperative on one wing:
  $N_T = 0 - ((-45 \text{ ft} \times -17000 \text{ lb}) + (-26 \text{ ft} \times -17{,}000 \text{ lb})) = -1{,}207{,}000$ ft-lb.

As the engine is considered aligned parallel to the X_b axis, the force F_y, being the sum of the Y_b (body) axis components of the lost thrust, spillage drag and lift at a distance ℓ_x in front of the y axis, is zero for this example and hence does no contribute to the thrust yawing moment N_T.
The propeller or spillage drag of an inoperative engine increases the thrust yawing moment N_T. To include this additional drag in the calculations, multiply N_T by 1.25 (fixed pitch), 1.1 (variable pitch), 1.15 (low bypass ratio) or 1.25 (high bypass ratio) (Ref. 7, page 218).

4.9. The determinant (Δ) of the matrices in § 4.7 is:

$$\Delta = (C_{\ell\beta} \times C_{n\delta a} \times C_{y\delta r}) + (C_{\ell\delta a} \times C_{n\delta r} \times C_{y\beta}) + (C_{\ell\delta r} \times C_{n\beta} \times C_{y\delta a}) - ((C_{y\beta} \times C_{n\delta a} \times C_{\ell\delta r}) + (C_{y\delta a} \times C_{n\delta r} \times C_{\ell\beta}) + (C_{y\delta r} \times C_{n\beta} \times C_{\ell\delta a}))$$

The solved equations for rudder deflection (δ_r), aileron deflection (δ_a) and angle of sideslip (β), while – for a turbojet – the coefficients $C_{\ell o}$, $C_{n o}$ and $C_{y o}$ are zero, were rearranged in a format with numerical factors, that are the sum of products of Coefficients, of mgϕ and of N_T, that can be used in a spreadsheet for calculating δ_r, δ_a and β respectively, in which weight mg (= W), bank angle ϕ, thrust yawing moment N_T and dynamic pressure q (airspeed) are the entry variables:

$$\delta_r = ((-C_{\ell\beta} \times C_{n\delta a}) + (C_{n\beta} \times C_{\ell\delta a})) \times mg\phi/qS\Delta + ((-C_{\ell\delta a} \times C_{y\beta}) + (C_{y\delta a} \times C_{\ell\beta})) \times N_T/qSb\Delta)$$

$\delta_a = ((-C_{\ell\delta r} \times C_{n\beta}) + (C_{n\delta r} \times C_{\ell\beta})) \times mg\phi/qS\Delta + ((-C_{\ell\beta} \times C_{y\delta r}) + (C_{y\beta} \times C_{\ell\delta r})) \times N_T/qSb\Delta)$

$\beta = ((-C_{\ell\delta a} \times C_{n\delta r}) + (C_{n\delta a} \times C_{\ell\delta r})) \times mg\phi/qS\Delta + ((-C_{\ell\delta r} \times C_{y\delta a}) + (C_{y\delta r} \times C_{\ell\delta a})) \times N_T/qSb\Delta)$

The matrix function MDETERM of a spreadsheet can also be used to solve the equations above.

4.10. The equations for δ_r, δ_a and β can be used during the preparation of flight tests to predict and identify the lateral and directional control and sideslip angle limitations that might occur during flight testing to avoid the loss of control while an engine is inoperative. The data is presented in line charts (§ 5 below).
In this paper, the equations are also used to calculate and present charts that show the effect of bank angle ϕ, weight (mg = W), thrust yawing moment N_T, airspeed ($q = ½ \rho V^2$) and altitude (air density ρ) on the *actual* (static) minimum control speed V_{MCA} (in § 5.9 below). The *actual* (static) V_{MCA} is the V_{MCA} that the pilot will experience in-flight after engine failure and that might be much higher than the AFM-published, standardized V_{MCA}.

5. Numerical example with control deflections and sideslip

5.1. In the examples presented below, the following lateral-directional stability derivatives of a 4-engine turbojet B707, DC-8 class airplane, determined with an aft cg location, were used:

$C_{\ell\beta}$	=	−0.0849	per radian	$C_{n\beta}$	=	0.1920	per radian	$C_{y\beta}$	=	−0.8660	per radian
$C_{\ell\delta a}$	=	0.0856	per radian	$C_{n\delta a}$	=	0.0106	per radian	$C_{y\delta a}$	=	0.0000	per radian
$C_{\ell\delta r}$	=	0.0218	per radian	$C_{n\delta r}$	=	−0.1660	per radian	$C_{y\delta r}$	=	0.3700	per radian
$C_{\ell o}$	=	0.0000	per radian	$C_{n o}$	=	0.0000	per radian	$C_{y o}$	=	0.0000	per radian

Table 3. Lat-dir stability derivatives of a sample Boeing 707/ DC-8 class airplane, aft cg.

5.2. After substituting these stability derivative data, wing area S (ft^2) and wing span b (ft) in the equations in § 4.9 above, leaving mgϕ, N_T and ρV variable, we get the following equations for δ_r, δ_a and β for this specific airplane:
(m in slug ($lb \cdot sec^2/ft$), g = 32,1740 ft/sec^2, mg = W in lb, ϕ in radians, N_T in ft-lb, ρ in slug/ft^3, V in ft/sec.
1 kt = 1.6878 ft/sec. At sea level, $\rho = \rho_0 = 0.0023769$ slug/ft^3).

δ_r (in radians) $= (0.001095\ mg\phi + 3.30 \times 10^{-5}\ N_T) / 0.5\ \rho V^2$

δ_a (in radians) $= (0.000626\ mg\phi + 5.57 \times 10^{-6}\ N_T) / 0.5\ \rho V^2$

β (in radians) $= (0.000912\ mg\phi + 1.41 \times 10^{-5}\ N_T) / 0.5\ \rho V^2$

Multiplying δ_r, δ_a and β (in radians) by 57.296 (= 360°/2π) returns the values in degrees.

5.3. With the stability derivatives presented above, the angle of sideslip β was calculated for different speeds, bank angles and weights, at sea level. Figure 5 below illustrates this predicted sideslip versus calibrated airspeed for four conditions (combinations of weight and bank angle) in which equilibrium of forces and moments is achieved (the airplane is in trim, i.e. the trim setting plus the manual control inputs in all three axes if the available trim controls are not adequate). In other words, the required rudder and aileron inputs do not exceed the maximum available travel of the controls.

5.4. When the wings are kept level (bank angle ϕ = 0), side force mgϕ is zero and sideslip β increases at decreasing airspeed until the side force due to β balances the side force due to rudder deflection $Y_{\delta r}$ whose yawing moment $N_{\delta r}$ is counteracting the asymmetrical thrust yawing moment N_T; precious energy is wasted (see forces in top of Figure 4 and in Ref. 3, § 2.7). At airspeeds below the airspeed at which β intersects with the side slip limit of ≈14° (120 KCAS), control might be lost, because the vertical fin stalls. As shown in Figure 5 below, at a speed of 225 KCAS still a sideslip of a few degrees remains depending on weight and bank angle.

Copyright © 2007 – 2023 AvioConsult

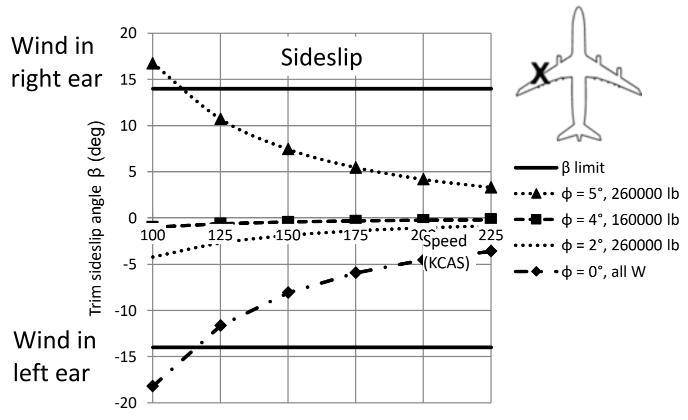

Figure 5. Predicted sideslip versus airspeed at given bank angle and weight, 4-engine turbojet, Sea Level, max. Thrust, engine #1 inoperative.

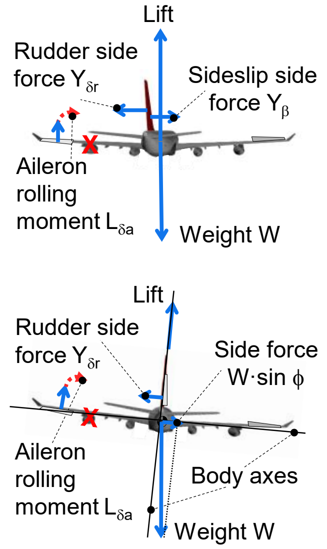

Figure 4. Side forces wings level and bank angle 5° in body axes.

5.5. The effect of bank angle is also shown in Figure 5. For a heavy airplane, a bank angle of 5° away from the failed engine, creates a large side force mgϕ (W·sin ϕ), resulting in a large (opposite) sideslip side force Y_β at low speed. Reducing the bank angle to 2.6° reduces sideslip β to near zero over the whole speed range. At low all-up weight, the sideslip β – hence the drag – at climb and cruise speeds, while maximum thrust is set on the operating engines, is only minimal if a small bank angle is maintained of 4°, as the data shows. Figure 4, bottom, shows the most important forces at a bank angle of 5°, low weight; lift does not have a side component in the body axes system.

5.6. The equations in § 5.2 were also used to calculate the predicted rudder and aileron deflections in Figure 6 below, for the same datapoints as in Figure 5, also for trimmed (static equilibrium) flight (i.e., with trims set and with additional control inputs, as required).

In the rudder deflection chart in Figure 6 below on the left side, the curved line for ϕ = 0 intersects with the maximum available rudder angle of −30° (= right rudder pedal), which equals wings-level V_{MCA} (120 kt), but due to the rudder pedal force limit of 150 lb, the actual wings-level V_{MCA} is a little higher, 125 kt. Below this speed, a static wings-level equilibrium cannot be maintained; control will be lost, unless a small bank angle away from the failed engine is attained and maintained. A bank angle of 5° away from the failed engine when at high weight requires zero rudder input, but the resulting sideslip, hence the drag, is very large, as shown in Figure 5. The rudder deflection limitation by the rudder ratio change system for airspeeds > 160 kt is not included; the rudder side force might be not be affected, though.

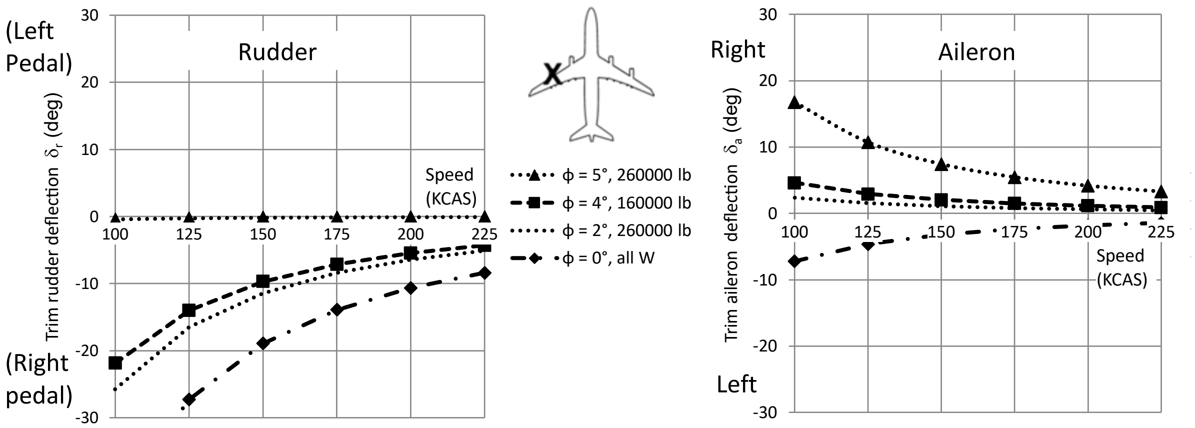

Figure 6. Predicted rudder and aileron deflections during trimmed flight, 4-engine turbojet, Sea Level, max. thrust, engine #1 inoperative.

5.7. In Figure 6 right, the predicted aileron deflection is presented that is required for achieving a static equilibrium, for maintaining lateral control. Maximum available aileron deflection of this airplane is 20°, while the maximum required aileron in the given speed range for the given bank angles and weights is ≤ 18°, hence the ailerons of this airplane do not restrict lateral control at low speed.

5.8. Although the figures are made using stability derivative data of a Boeing 707/DC-8 class airplane, because the author did not have lateral-directional stability derivatives of other types, similar figures apply to all multi-engine airplanes.
Most critical for small multi-engine airplanes is the remaining performance. If a pilot, after engine failure, does not maintain a small bank angle away from the failed engine and rudder to maintain straight flight, the sideslip is not minimal and performance is lost. If a small bank angle is being maintained, it could be that just a small rate of climb remains, in which case it may take up to 30 minutes to reach a safe altitude, where the speed can be increased in level flight to make safe turns.
Many airplanes that suffered from an engine failure during climb or cruise didn't make it to the shore or to an airport. A small bank angle – less drag, longer range – at the proper maximum range or drift down speed provides for an increased engine-inoperative range and might have saved lives.

5.9. V_{MCA} is the minimum airspeed to be observed in anticipation of an engine failure. When one engine of a 4- or more-engine airplane failed or is inoperative in-flight, V_{MCA2} is the minimum airspeed to be observed in anticipation of a second engine to fail. V_{MCA2}, the minimum control speed when two engines on the same wing are inoperative (is worst case), is much higher than $V_{MCA\ (1)}$. Regrettably, civil regulations do not require V_{MCA2} to be determined anymore (only V_{MCL2}, which is of no use), but military regulations still do. Data for a two engines inoperative case are therefore provided below in Figure 7, without further comments, except that effects of the rudder ratio change system, when the airspeed > 160 kt, are not included here either.

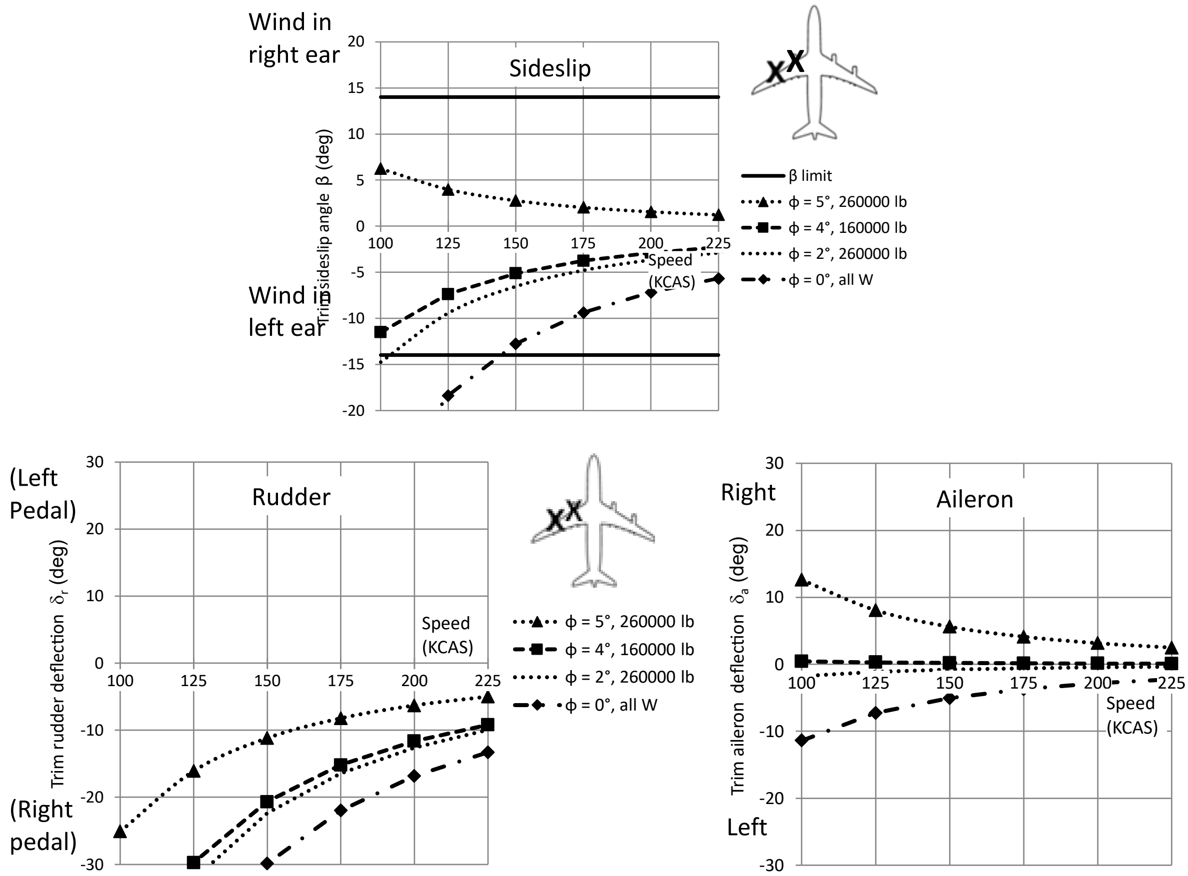

Figure 7. Predicted sideslip, rudder and aileron deflections during trimmed flight, 4-engine turbojet, Sea Level, max. thrust, engines #1 and #2 inoperative.

Copyright © 2007 – 2023 AvioConsult

6. The effect of bank angle and weight on V_{MCA}

6.1. As announced in § 4.10 above, the derived equations can also be used to calculate and show the effect of bank angle and weight on V_{MCA} in a way that is more comprehensible for flight operations. As the pilot controls the bank angle, it is very important for pilots to understand the effect of bank angle and Weight on V_{MCA}. Applying this knowledge will prevent the loss of control after engine failure.

6.2. In accordance with the definition of V_{MCA} in FAR/ CS 23.149 and 25.149 (Refs 1, 2), the lowest airspeed, at which one of the control surface maxima of rudder (δ_r) or ailerons (δ_a) is reached while one engine is inoperative, is minimum control speed V_{MCA}. In addition, the civil regulations allow a maximum rudder force of 150 lb (667 N) and a maximum roll control force of 25 lb (112 N).
Military specifications require that roll control shall not exceed the roll stick/wheel control force limit (20 lb) or 75% of the control power available to the pilot, leaving room for countering gusts, transient effects and to maneuver; the maximum pedal force is 180 lb.
These stick/wheel and pedal force limitations are not included in the equations, but are subject of flight-testing; the V_{MCA} determined in-flight may therefore be higher than the calculated or predicted V_{MCA}.
There must of course remain a reason for conducting actual V_{MCA} flight testing…
For the purpose of this paper, and of the paper *Airplane Control and Analysis of Accidents after Engine Failure* (Ref. 3) though, the calculated prediction data, even without the availability of control force data, will provide insight in the mostly not well-known effect of bank angle and weight on V_{MCA}.

6.3. The equations of § 5.2 can be rearranged to be solved for the speed at which either the maximum available deflection of rudder or ailerons is reached, or the maximum allowable angle of sideslip is reached for bank angles ϕ between −15° and +15°. For the sample airplane, these limits are: δ_r is max. 30°, δ_a is max. 20°, and β is max. 14° (to avoid the fin to stall). The derived equations are:

$$V_{max.\ \delta r}\ (kt) = (1/1.6878) \times (\sqrt{((0.001095\ mg\phi + 3.30 \times 10^{-5}\ N_T) / (\delta_{r\ max} \times 0.5\ \rho))})$$

$$V_{max.\ \delta a}\ (kt) = (1/1.6878) \times (\sqrt{((0.000626\ mg\phi + 5.57 \times 10^{-6}\ N_T) / (\delta_{a\ max} \times 0.5\ \rho))})$$

$$V_{max.\ \beta}\ (kt) = (1/1.6878) \times (\sqrt{((0.000912\ mg\phi + 1.41 \times 10^{-5}\ N_T) / (\beta_{max} \times 0.5\ \rho))})$$

Weight mg (W) should be entered in lb; $\delta_{r\ max}$, $\delta_{a\ max}$, β and ϕ in radians (360° = 2 π radians).
At sea level $\rho = \rho_0 = 0.0023769$ slug/ft^3, 1 kt = 1.6878 ft/sec.

Again, for deriving these equations, the small bank angle assumption ($\sin\phi = \phi$ (in radians)) was used: ϕ should be less than 5 degrees. However, the difference between $\sin\phi$ and ϕ (in radians) when ϕ = 15° is only 0.00298. For the purpose of only illustrating the effect of bank angle on V_{MCA} it is considered acceptable to show data for bank angles up to ±15°, the objective being to show the effect of bank angle and weight, rather than exact numbers.

6.4. The *actual* V_{MCA}, while bank angle and/ or weight vary and the thrust N_T is maximum, is the highest speed of either one of the speeds $V_{max.\ \delta r}$, $V_{max.\ \delta a}$ and $V_{max.\ \beta}$ that are calculated using the three equations above, because then one of the physical control surface deflection limits is reached, or the angle of sideslip β is at the limit. This highest speed is plotted as the *actual* V_{MCA} for the bank angle range −15° to +15° in figures below. The airspeed while maintaining the corresponding bank angle should not be decreased below this speed, or the airplane will be out of control because there is either no rudder or aileron control power left for maintaining a static equilibrium (trimmed flight), or the increased angle of sideslip causes the vertical fin to stall. As mentioned above, rudder and aileron control forces are not included in the calculation, which might lead to a slightly higher *actual* V_{MCA} during flight testing.

6.5. The equations presented above can be entered in a spreadsheet to solve and plot the *actual* V_{MCA} for several weights (mg) and bank angles (ϕ) into or away from the inoperative engine(s), together with the rudder and aileron control inputs (δ_r and δ_a), and with the side slip angle. These data are presented in the figures below, for One Engine Inoperative (OEI) and Two Engines Inoperative (TEI).

Copyright © 2007 – 2023 AvioConsult

6.6. **One Engine Inoperative**. For the case that engine #1 is inoperative, a (lost) thrust yawing moment N_T of −765.000 ft-lb was used in equations of § 5.2. Two charts were made, on the left side to read V_{MCA} using weight and bank angles, on the right side to read V_{MCA}, aileron and rudder deflection and sideslip angle for the bank angle range –15° to +15° and for both minimum and maximum weight.

6.7. The left-hand chart in Figure 8 below shows the influence of weight on the predicted (*actual)* V_{MCA} for several bank angles for this sample airplane. With the wings level (ϕ = 0°), weight has no effect on V_{MCA} because side force W·sin ϕ = 0 (mgϕ in § 6.3); the *actual* V_{MCA} for all weights with the wings level is ≈119 kt. With a 5° bank angle away from the failed engine, the standardized, low weight V_{MCA} is lower and decreases initially with increasing weight, but then increases. However, the predicted V_{MCA} for this bank angle (which is the AFM-published V_{MCA}) is lower than the stall speed V_S for all weights (but only during straight flight with a bank angle of 4° away from the inoperative engine).
A bank angle into the inoperative engine increases V_{MCA} considerable with increasing weight. The airspeed to avoid the loss of control needs to be much higher. The V_{MCA} for several bank angles at a weight of 160,000 lb coincide with the light weight V_{MCA} data in the right-hand chart.

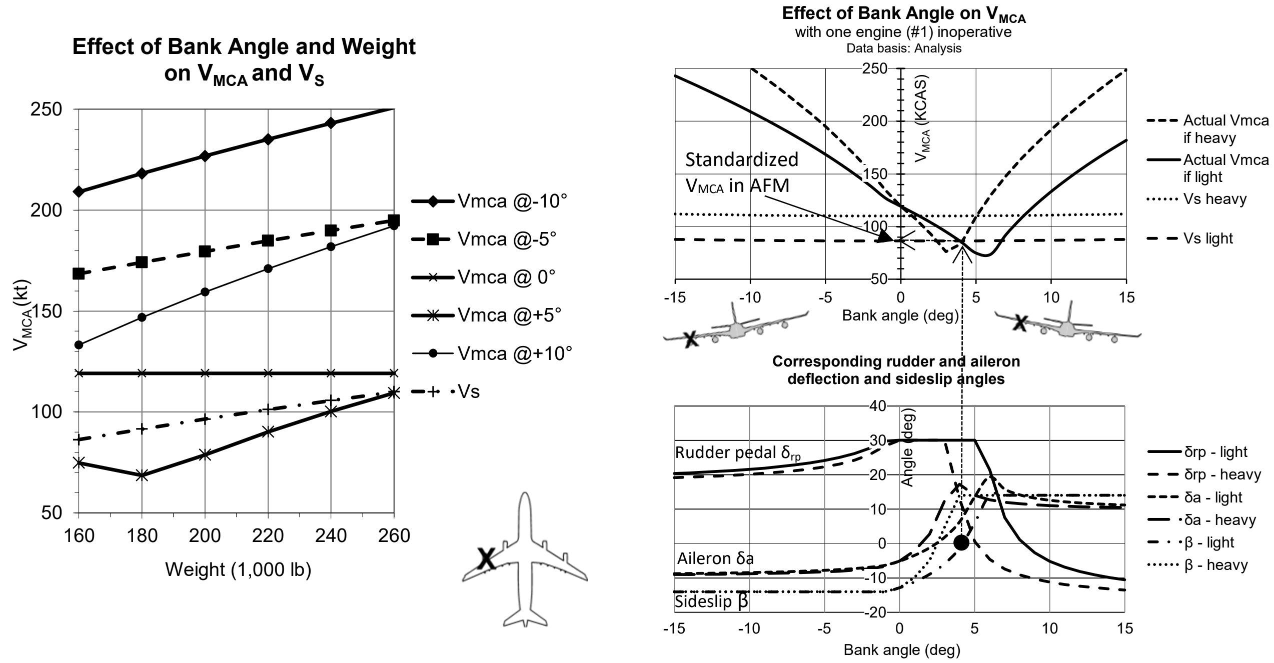

Figure 8. Predicted effect of bank angle and weight on V_{MCA} during trimmed flight, 4-engine turbojet, Sea Level, max. thrust, engine #1 inoperative.

6.8. The right-hand chart shows the effect of bank angle on V_{MCA} and control deflections, and the resulting sideslip. Rather than the negative rudder surface deflection δ_r, which is limited by the rudder ratio system at airspeeds > 160 kt for this sample airplane, the positive rudder pedal deflection (δ_{rp}) is shown for clarity, but might not be to scale.
V_{MCA} is determined at a bank angle at which the sideslip angle β is zero degrees when the airplane is low weight, the worst-case weight for V_{MCA}, in this case 3° (dotted line). Then the predicted V_{MCA} is ≈95 kt, but very close to the stall speed V_S. The chart shows that V_{MCA} is predicted to be below V_S at bank angles between 4° and 7°, which is favorable, because the airplane will be controllable down to the stall when such a bank angle is being maintained. The stall warning will sound before the airspeed is decreased to V_{MCA}, but not when the wings are kept level (or at larger bank angles). Then the *actual* V_{MCA} is ≈20 kt higher than the published V_{MCA} and the sideslip as large as 13°; the large drag decreases the climb performance. This V_{MCA} increase is not made clear in most AFMs, but is important to know in case an engine fails during takeoff. The *actual* V_{MCA} might even increase above the calculated takeoff speeds.
Please also notice the rudder reversal required for equilibrium of side forces and yawing moments, i.e.,

Copyright © 2007 – 2023 AvioConsult

for maintaining control when ϕ > 5° into the good engine(s), because of the increased sideslip due to increased W·sin ϕ; the rudder is now required to reduce the increased yawing moment due to sideslip to maintain balance with N_T. Many more variables have influence on V_{MCA}, refer to Ref. 3, § 4 for details.

6.9. **Two Engines Inoperative.** Although FAA and EASA Regulations (Refs 1, 2) do no longer require a V_{MCA2} to be determined for 4- of more-engine airplanes, but only a V_{MCL2} (which the author finds incomprehensible), a V_{MCA2} still exists for these airplanes – and accidents due to the loss of thrust happen. Pilots not made aware might get into real trouble controlling their airplane while two engines are inoperative on, or departed from the same wing. Therefore, V_{MCA2} data are presented here as well, as also in § 5.9. For this case, a (lost) thrust yawing moment N_T of −1,207,000 ft-lb was used in the equations of § 5.2. Again, two charts were made, on the left side to show V_{MCA} using weight and bank angles, on the right side to show the actual V_{MCA}, aileron and rudder (pedal) deflections and sideslip angle for the bank angle range –15° to +15° and for minimum and maximum weight.

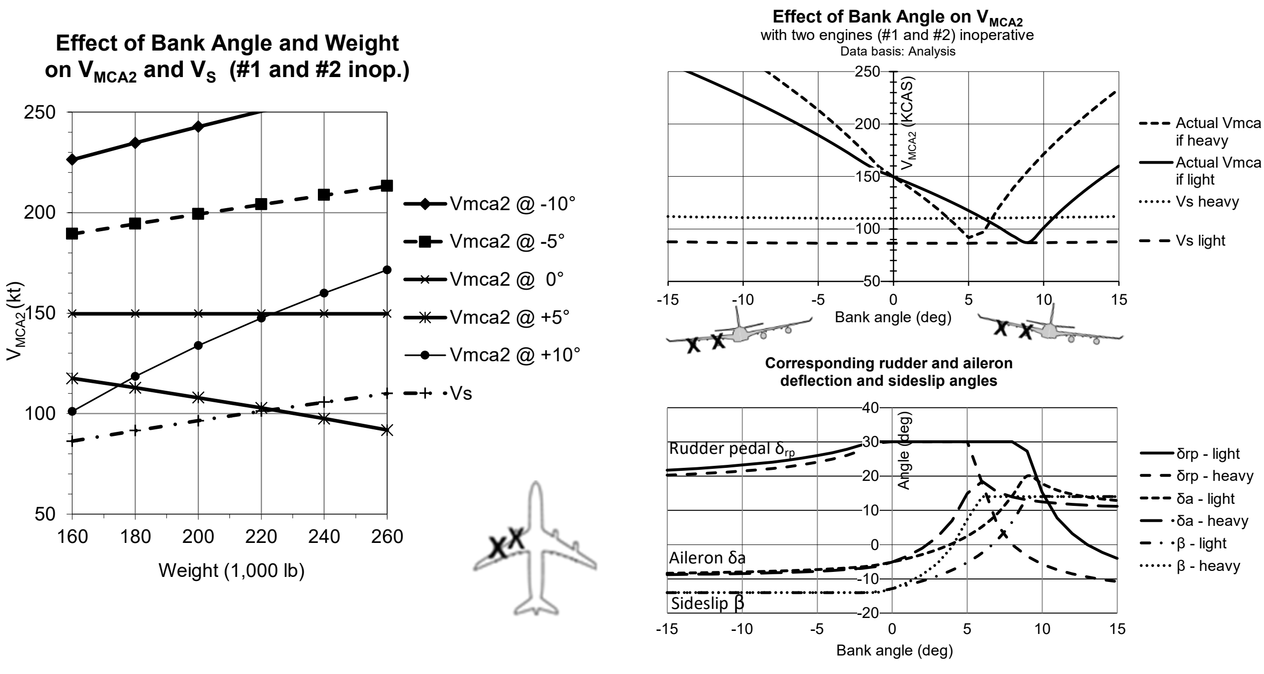

Figure 9. Predicted effect of bank angle and weight on V_{MCA2} during trimmed flight, 4-engine turbojet, Sea Level, max. thrust, engines #1 and #2 inoperative.

6.10. The reader might notice that the required airspeeds for maintaining a static equilibrium, for not losing control, are much higher than for one engine inoperative. Banking the airplane for a turn with only a small bank angle, while the remaining engines are set to provide maximum thrust, increases the *actual* V_{MCA} surprisingly high. If the airspeed is not increased first, control will be lost, by definition. This happened with a B747 that lost two engines and crashed in the suburbs of Amsterdam on Oct. 4, 1992. Neither the pilots, not the accident investigators were aware of the characteristics of V_{MCA}, and of the conditions/restrictions that come with it. Physics is everywhere.

7. Conclusions and recommendations

7.1. The results of the analysis presented in this paper show that bank angle and weight obviously have great effect on the *actual* V_{MCA} of a multi-engine airplane, being the V_{MCA} that the pilot will experience in-flight. It is proven that the (standardized) V_{MCA} that is presented in the Airplane Flight Manual is valid only if the same bank angle is being maintained that was used to determine V_{MCA}, i.e., during unaccelerated, 1g, constant heading (straight) flight, and definitely not during turns. Any other bank angle, whether this is zero, into the inoperative engine, or larger than 5° into the operative engine will result in a higher *actual* V_{MCA} and might lead to loss of control if the asymmetrical power setting is. Not reversing the rudder at bank angles >6° into the operative engine will yaw the airplane towards the ground.

Copyright © 2007 – 2023 AvioConsult

7.2. Minimum control speed $V_{MC(A)}$ is not the lowest speed at which "control" can be maintained, but is only the lowest speed at which straight flight can be maintained with maximum rudder and/or aileron deflection, when maximum asymmetrical thrust is set, provided a small bank angle is being maintained away from the inoperative engine.

7.3. In Airplane Flight Manuals, the bank angle that was used to determine V_{MCA} is a condition that is required for the published, standardized V_{MCA} to be valid, and should therefore be published with the V_{MCA} data. In engine emergency procedures pilots should be reminded to maintain that bank angle, hence to maintain straight flight, until reaching a safe altitude (Ref. 3). If a small bank angle away from the inoperative engine is used to determine V_{MCA}, which most often will be the case for small multi-engine airplanes to reduce the drag to a minimum therewith increasing the remaining climb performance, it is recommended to consider to publish the V_{MCA} for zero bank angle as well, because takeoff safety speeds are calculated using the published V_{MCA} which is usually 8 to 30 kt, depending on the airplane type, lower than wings-level V_{MCA}. If after an engine failure during takeoff the wings are kept level, the *actual* V_{MCA} is a lot higher, as is the drag. Then the safety margin above V_{MCA} as required in regulations is much smaller or no longer available and decreases even further during turns (at low speed). Many airplanes crashed during takeoff when an engine failed and a turn was initiated soon thereafter at too low a speed.

7.4. The figures calculated in this paper are used in Ref. 3. Additional factors that have influence on V_{MCA}, a detailed explanation of the presented figures and data, and the consequences of not maintaining the favorable 3 to 5° bank angle during takeoff when an engine fails, are also presented in Ref. 3, which can be used to improve V_{MCA} definitions and engine emergency procedures in Airplane Flight Manuals, as well as to improve the investigation of accidents after engine failure, as presented in several examples in Ref. 3.

References

1. Federal Aviation Regulations Part 25.149 or Part 23.149, https://www.ecfr.gov/cgi-bin/text-idx?SID=a368e3764d61c98423b17629b48e8223&mc=true&tpl=/ecfrbrowse/Title14/14CIsub-chapC.tpl.
2. EASA Certification Specifications CS-23.149 or CS 25.149, https://www.easa.europa.eu/en/document-library/certification-specifications.
3. Harry Horlings, *Airplane Control and Analysis of Accidents after Engine Failure*. Avio*Consult*, June 2005, https://www.avioconsult.com/downloads.htm, paper #3.
4. USAF Test Pilot School, *Flying Qualities Testing, Failure State Testing, Chapter 32.* Edwards Air Force Base, CA, USA, ≈2001.
5. *Flying Qualities Textbook*, Volume II, Part 2, 1986 (Chapter 11 = Asym power). USAF Test Pilot School, Edwards Air Force Base, CA, USA. USArchive/DTIC ADA170960, https://ia801001.us.archive.org/17/items/DTIC_ADA170960/DTIC_ADA170960.pdf.
6. USNaval Test Pilot School, *Fixed Wing Stability and Control* (Chapter 6 = Asym. power), FTM-103, Patuxent River, MD, USA, http://www.usntpsalumni.com/Resources/Documents/USNTPS_FTM_103.pdf
7. Dr. Jan Roskam, *Airplane Design; Stability and Control During Steady Straight Flight*, Chapter 4, Kansas University, DAR Corporation, Kansas.
8. FAA Flight Test Guide Part 25 Airplanes (Controllability and Maneuverability, including V_{MCA} testing, in § 5.4), https://www.faa.gov/documentLibrary/media/Advisory_Circular/25-7D.pdf.
9. FAA Flight Test Guide Part 23 Airplanes (Controllability and Maneuverability, including V_{MCA} testing, in Section 4.4 § 23.149), https://www.faa.gov/documentLibrary/media/Advisory_Circular/AC_23-8C.pdf.
10. EASA Flight Test Guide (V_{MCA} testing on page 257), https://www.easa.europa.eu/sites/default/files/dfu/agency-measures-docs-certification-specifications-CS-23-CS-23-Amdt-3.pdf.

Copyright © 2007 – 2023 AvioConsult